

## Culture and cooperation in a spatial public goods game

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We study the coevolution of culture and cooperation by combining the Axelrod model of cultural dissemination with a spatial public goods game, incorporating both noise and social influence. Both participation and cooperation in public goods games are conditional on cultural similarity. We find that a larger “scope of cultural possibilities” in the model leads to the survival of cooperation, when noise is not present, and a higher probability of a multicultural state evolving, for low noise rates. High noise rates, however, lead to both rapid extinction of cooperation and collapse into cultural “anomie,” in which stable cultural regions fail to form. These results suggest that cultural diversity can actually be beneficial for the evolution of cooperation, but that cultural information needs to be transmitted accurately in order to maintain both coherent cultural groups and cooperation.

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### I. INTRODUCTION

How a collective structure emerges from microlevel processes is a critical question not only in the natural sciences but also the social sciences. In particular, the emergence of cultural diversity and the evolution of cooperation have been two questions of long-standing interest in a variety of fields including evolutionary biology, economics, social psychology, and sociophysics. The emergence of cultural diversity has often been examined using Axelrod’s model of cultural dissemination [1], whereas the evolution of cooperation has been examined within the framework of evolutionary game theory [2–5], to which Axelrod also made seminal contributions [2,3,6,7]. Nevertheless, the coevolution of cultural diversity and cooperation began to be investigated only recently [8–12, e.g.]. At first glance, the coevolution of culture and cooperation, without requiring direct [2] or indirect [13,14] reciprocity, may seem like a simple question of a tag-based evolution of cooperation—if culture is seen as a “tag” like a “green beard” [7,15], a strategy to conditionally cooperate with another agent that shares a similar culture (i.e., a tag) should be evolutionarily stable [7]. Culture and cooperation, therefore, should unproblematically coevolve.

Despite the elegance of this formulation, it hides some complexities. One is the temporal dynamics of the coevolution of culture and cooperation. As Valori *et al.* [12] note, cooperation needs to evolve over a relatively short time span, whereas culture evolves over a much longer time scale. Their solution then was to model them separately using different models; however, no model has been developed that can shed light on the puzzle of differential time scale—how can culture and cooperation coevolve despite the difference in time scale? Another is the question of cultural diversity and cooperation. The tag-based evolution of cooperation seems to suggest that cultural similarity may be a key to the evolution of cooperation. However, recent research on diversity and cooperation [16–24] suggests that in fact diversity is beneficial

for the evolution of cooperation. This raises a question, which is not only theoretically important but also potentially of social significance in light of the cultural diversity in the contemporary social context. That is, what is the role of cultural diversity in the coevolution of culture and cooperation? Is it cultural similarity or diversity that is beneficial for the evolution of cooperation?

We will show that Axelrod’s model of cultural dissemination in combination with a mechanism of tag-based conditional cooperation can shed light on these twin questions of time scale and cultural diversity.

#### A. Axelrod’s model of cultural dissemination

In the Axelrod model [1] the dynamics of cultural diffusion are modeled on a (fully occupied) bounded lattice, based on the assumptions of *homophily* (agents prefer to interact with similar others) and *social influence* (agents become more similar in their cultural attributes when they interact). The cultural attributes of agents are modeled as an  $F$ -dimensional vector, each element (feature) of which can take one of  $q$  possible values (traits). Then the cultural similarity of two agents is the number of features they have in common. Depending on the initial diversity, controlled by the vector dimension  $F$ , and particularly the number of trait values  $q$ , or the *scope of cultural possibilities* as Axelrod [1] describes it, the model converges to either a *monocultural state*, in which all agents have the same culture, or a *multicultural state* in which regions of agents with the same culture form, with agents on the boundary between two regions having no feature in common.

The Axelrod model has subsequently been extended in a variety of ways, including the introduction of “mass media” or other external cultural influence [25–29], complex networks [30,31], coevolving networks [32,33], and agent migration [34,35]. Two important extensions to the model are nondyadic interactions [36–38] and noise [32,36,39,40].

Both noise and nondyadic interactions (that is, interactions between more than two agents, which we term *multilateral influence*) are explored in Flache and Macy [38]. Multilateral influence can create qualitatively different results in Axelrod

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models compared to dyadic influence [36–38], as can noise [30,38,41].

### B. Evolution of cooperation

The evolution of cooperation is often examined in terms of the public goods game [42–44], which can be viewed as an extension of the well-known prisoner’s dilemma to more than two players. In this game, there is a common pool to which players may either contribute (cooperators) or not (defectors). The contributions are then multiplied by some factor, and the pool divided equally among the players. Hence the dilemma, or “tragedy of the commons” [45], arises as any individual is better off by not contributing (defecting), but the optimal outcome as a whole is for all players to contribute (cooperate). A variety of factors influence the evolution of cooperation in the iterated version of such games [2,3,44], including conformity [46], tolerance towards defectors [47], voluntary participation [43], conditional cooperation [48], various punishment and reward strategies [49–53], group synergy or discount effects [54], group reputation memory [55], strategy updating heuristics [56], migration [57], population density [58], or social exclusion (removing the benefit from free riders) [59]. The game is often played on a lattice or other structured population, giving it a spatial aspect [60], which was already found to give an entirely new dimension to the prisoner’s dilemma game [61]. The population structure then becomes a relevant factor [62–65].

Another potentially important factor in the public goods game is the group size [60,66–73]. Some properties, such as the pool multiplier  $m$ , are properties of the group, and some, such as the marginal per capita return (MPCR), relate to the individual. The MPCR (which we will denote  $\beta$ ) is defined as  $\beta = \frac{m}{N}$  where  $N$  is the group size. Therefore either  $m$  or MPCR can be held constant when the group size changes, but not both [73].

Just as in the cultural diffusion model, interactions between more than two players in evolutionary games can produce results which cannot be generated by pairwise interactions only [60]. Similarly, the introduction of noise can also produce qualitatively different results in evolutionary games [41,68]. Coevolution of other factors, such as networks or migration [74,75], strength of influence in strategy adoption [76], as well as migration, reputation, and age [77], have also been incorporated into evolutionary games.

Recently, there has been a lot of work examining the role of diversity in the evolution of cooperation, including Perc and Szolnoki [16], examining the role of payoff scaling factors in promoting cooperation in a spatial prisoner’s dilemma game. Generally, it is found that various kinds of diversity or heterogeneity promote cooperation [17], in particular in the spatial public goods game. This is shown for heterogeneity of pool multiplication factor [18,19]; having two types of agents, with different strategy adoption rules [20]; assortment by reputation, where reputation is a dynamic scaling factor on the update function [78]; heterogeneity of contribution to the pool based on social ties [22]; diversity of strategies [23]; and resource heterogeneity [24]. An exception is Perc [79], where it is shown that a uniform distribution of payoffs is found to be

more effective at promoting cooperation than an exponential distribution, in the context of group interactions.

### C. Coevolution of culture and cooperation

Some work examines the evolution of both cooperation and culture. Allison [80] suggests some mechanisms of cultural transmission that may result in “beneficent behavior” (such as cooperation). These are cultural analogs of kinship theory [81,82], that is, that norms will evolve to direct beneficent behavior towards close (cultural) relatives, or towards those who show certain cultural markers. Our model adopts this theoretical mechanism: the probability of cooperation for a “cooperator” in the public goods game proportional to agents’ cultural similarity. Also applying this theory to a variation of the Axelrod model is the work of Heinrich *et al.* [9], in which agents on a lattice must cooperate to work on jobs, which require a minimum number of cooperating agents to complete. Agents only cooperate if their cultural similarity is above an “altruism threshold,” and the effect of the value of this threshold on successful job completion is analyzed.

An important advance in the theory of evolution of cooperation is the idea of tag-based cooperation [7], in which it was shown that evolution can emerge without requiring reciprocity or memory, when agents have some detectable trait, or tag, and cooperation is conditional on agents having sufficiently similar tags. The model allows the formation of clusters of agents with similar tags, which can allow cooperation to evolve and survive without reciprocity.

Gargiulo and Ramasco [10] examine the influence of opinion dynamics [83] on an evolutionary public goods game, finding that new features emerge that are not present in the opinion dynamics or evolutionary game alone. Zhang [11] combines a spatial prisoner’s dilemma game with a tag which resembles a culture vector in the Axelrod model, however the trait values are binary only, and social influence consists of copying the entire tag. Two agents can interact only when their tag similarity is above a given tolerance threshold. It is found that cooperation does not die out, but forms clusters of cooperating agents, only for certain intermediate values of the threshold.

By combining the Axelrod model with a variation of the Schelling model of residential segregation [84,85] (as also done in Gracia-Lázaro *et al.* [86]), and incorporating a prisoner’s dilemma evolutionary game, Helbing *et al.* [8] show that, on their own, neither imitation of best strategy nor migration to a more favorable location promotes cooperation. However, when both are combined, cooperators and defectors self-organize into cooperative clusters which are robust to randomness.

Valori *et al.* [12] examine both cooperation and the evolution of cultural diversity, using two separate models: an extended Cont-Bouchaud model [87] for coordination of individuals’ choices and the Axelrod model, respectively. It is shown that it is a particular property of the distribution of culture vectors, namely, ultrametricity [88] (although, see also Stivala *et al.* [89], Babeanu *et al.* [90]), that allows both cooperation on short time scales and cultural diversity on long time scales. Importantly, however, the models of culture and cooperation are completely separate: the choice variable used in the modified Cont-Bouchaud model is *not* a cultural

variable (that is, not part of the Axelrod model) as it is specifically assumed that culture evolves over a longer time scale [12, S.I., p.5].

In this work we combine an Axelrod model of cultural dissemination (including multilateral influence and noise) with a spatial public goods game in which the probability of both participation and cooperation are based on cultural similarity. The tag used for conditional cooperation is thus the culture vector used in the Axelrod model. Because participation in a public goods game is conditional on cultural similarity, the number of players in each game is a dynamic quantity. Similarly, since the probability of a “cooperating” agent actually cooperating in a game is determined by cultural similarity, cooperation is conditional and its probability is also a dynamic quantity which evolves along with the dynamics of the Axelrod model. In this way the model allows us to examine the coevolution of culture and conditional cooperation.

## II. MODEL

In the Axelrod model, each agent on the fully occupied  $L \times L$  lattice has an  $F$ -dimensional culture vector, each element of which takes an integer value between 1 and  $q$  (inclusive). The cultural similarity of two agents is the number of features they have in common. If we denote element  $i$  of the culture vector of agent  $a$  by  $v_{ai}$ , then the cultural similarity  $0 \leq c(a,b) \leq 1$  of two agents  $a$  and  $b$  is a normalized Hamming similarity

$$c(a,b) = \frac{1}{F} \sum_{i=1}^F \delta_{v_{ai}, v_{bi}} \quad (1)$$

where  $\delta_{x,y}$  is the Kronecker delta function.

An agent can interact with its neighbors, defined as the von Neumann neighborhood, that is, the four surrounding cells on the lattice. This can be extended to larger von Neumann neighborhoods by increasing the radius, that is, extending the neighborhood to all cells within a given Manhattan distance.

Initially, the agents are assigned uniform random culture vectors. At each step of the model, a focal agent  $a$  is chosen at random. With probability proportional to their cultural similarity (the number of features on which they have identical traits), it interacts with a randomly chosen agent  $b$  from its neighborhood. This interaction results in a randomly chosen feature on  $a$  whose value is different from that on  $b$  being changed to  $b$ 's value. This process is repeated until no more change is possible, because all agents' neighbors have either identical or completely distinct (no features in common, so no interaction can occur) culture vectors.

In Flache and Macy [38], the model is extended to include multilateral influence and noise. Multilateral influence means that rather than the focal agent  $a$  interacting with only one of its neighboring agents in a single step it interacts with multiple neighboring agents. Hence, a set  $S$  of influential agents is built by considering each agent in the neighborhood of  $a$ , and adding it to  $S$  with probability proportional to its cultural similarity to  $a$ . Then a random feature on which change is possible (that is, one on which at least one trait different from  $a$ 's is shared by at least as many agents in  $S$  as  $a$ 's current trait) is chosen and  $a$  adopts the modal value of that feature among the agents in  $S$ . If there is more than one modal value, one is randomly chosen,

although giving preference to the focal agent's existing value if it is modal. Noise is introduced at two points: selection error (at rate  $r'$ ) and perturbation (at rate  $r$ ). Selection error operates on the decision on whether to add an agent to the influential set  $S$ . An agent is added to  $S$  with probability  $p_{ab}$  equal to the proportion of  $F$  features on which  $a$  and  $b$  have identical traits, that is  $p_{ab} = c(a,b)$ . Selection error means that with probability  $r'$ , this decision is reversed (in either direction; it is not added when it otherwise would have been, and added when it otherwise would not have been). Perturbation error is an additional step after the existing interaction steps: a feature is randomly chosen and with probability  $r$  is changed to a randomly chosen value, as in Klemm *et al.* [30,91]. Flache and Macy [38] assume a single noise rate  $r = r'$ , as do we.

We extend the model further to include multiple public goods games played by the interacting agents. To this end, each agent, in addition to a culture vector, has a strategy trait, which has one of the two values *cooperate* or *defect*. Initially, each agent is assigned one of the two strategies at random. An interaction between a focal agent and its neighbors is now defined as (potential) participation in a fixed number  $k$  ( $k = 5$  in all experiments described here) of public goods games. Note that the “cooperate” strategy does not mean the agent will always cooperate: cooperation is conditional on cultural similarity.

Along with the focal agent  $a$ , each agent  $b$  in the neighborhood of  $a$  participates in public goods game  $j$  ( $1 \leq j \leq k$ ) with probability  $p_{ab} = c(a,b)$ , i.e., the cultural similarity of  $a$  and  $b$ . Let the participating agent be called  $i$  (which can either be the focal agent  $a$  or any of the agents in its neighborhood). If  $i$  is a conditionally cooperating agent (that is, its strategy trait is “cooperate”), then with probability  $p_{ai} = c(a,i)$ , the cultural similarity of  $a$  and  $i$ , it contributes cost  $\alpha$  (without loss of generality,  $\alpha = 1$ , as agents have an infinite “budget” in our model to avoid effects caused by budget exhaustion) to the public goods game pool for game  $j$  and we write  $s_{ij} = 1$ . If  $i$  is a defector, then it contributes nothing, and we write  $s_{ij} = 0$ . Note that the focal agent  $a$ , if it is a conditional cooperator, will therefore always have  $s_{ij} = 1$ . Therefore for game  $j$  each agent  $i$  receives payoff (or benefit)

$$B_{ij} = m \frac{N_{jC}\alpha}{N_j} - s_{ij}\alpha \quad (2)$$

where  $N_{jC} = \sum_i s_{ij}$  is the number of contributing agents,  $N_j$  is the total number of participating agents in public goods game  $j$ , and  $m = \beta N_j$  is the public goods game multiplier, with  $0 < \beta < 1$  the marginal per capita return. The payoff [Eq. (2)] can then be simplified to

$$B_{ij} = \beta N_{jC}\alpha - s_{ij}\alpha \quad (3)$$

eliminating the group size as a factor. In this way, the individual property MPCR is held constant as the group size varies so that the payoff does not depend directly on group size [73]. The total payoff for agent  $i$  over all the games is  $B_i = \sum_j B_{ij}$ .

The focal agent's strategy and culture vector are then updated according to the Fermi update rule, which determines a probability of the update occurring proportional to the difference in payoff between two agents, as described in, for example, Perc *et al.* [60]. One of the participating agents  $b$  is chosen at random, and the focal agent adopts its strategy with

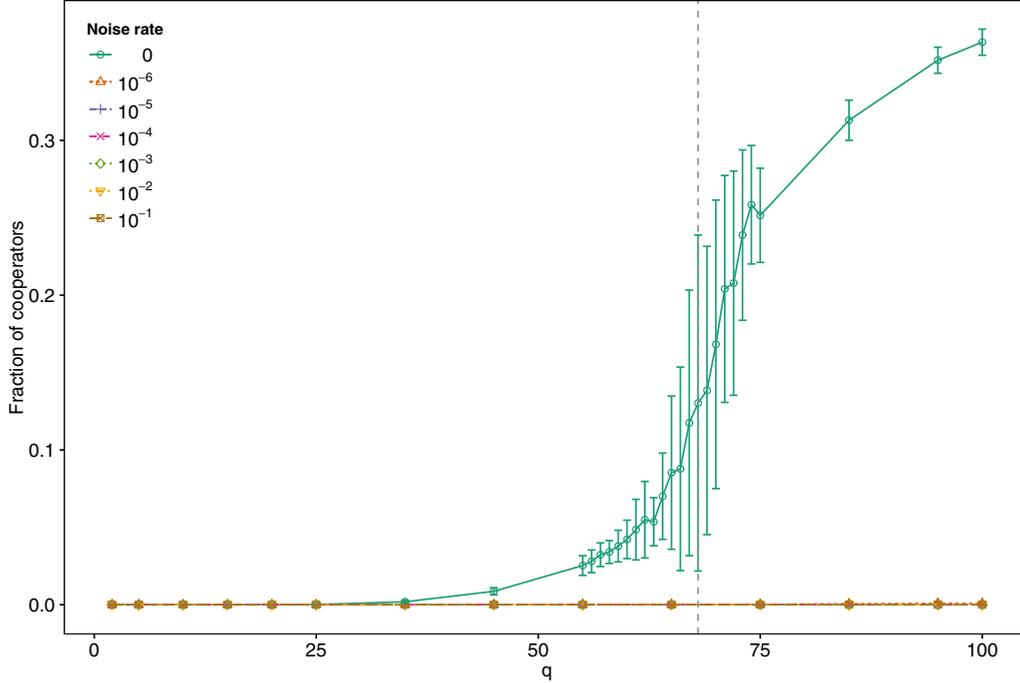


FIG. 1. Fraction of agents which are (conditional) cooperators after  $10^9$  steps as a function of  $q$  for different noise rates. Lines are shown purely as an aid to the eye as  $q$  is an integer. The dashed vertical line shows the critical value of  $q$  (for zero noise).

probability  $P(\sigma_b \rightarrow \sigma_a) = P_f(B_a, B_b)$ , where  $\sigma_x$  indicates the strategy trait of agent  $x$ . In addition, the focal agent adopts the trait value of a randomly chosen feature of a randomly chosen participating agent  $i$  with probability  $P(v_{ij} \rightarrow v_{aj}) = P_f(B_a, B_i)$ , where  $1 \leq j \leq F$  is a randomly chosen feature such that  $v_{ij} \neq v_{aj}$ , if such a feature exists. Here  $P_f$  is the Fermi function:

$$P_f(x, y) = \frac{1}{1 + \exp[(x - y)/K]} \quad (4)$$

where  $K$  is a “temperature-like” parameter quantifying a level of uncertainty in strategy or trait adoption. In the limit  $K \rightarrow 0$ , the strategy or cultural trait of the agent with the superior payoff will always be chosen, while in the limit  $K \rightarrow \infty$ , the probability of adopting the strategy or cultural trait of the agent with higher payoff is  $1/2$  [60,68,92–95].

Note that this rule for updating the culture is different from that described in Flache and Macy [38]: rather than choosing the modal trait value among the participating agents, the focal agent instead adopts the trait value of a randomly chosen participating agent with probability  $P(v_{ij} \rightarrow v_{aj})$ , a quantity which depends on the payoffs of the focal and one other agent (and hence indirectly on the cultures and strategy traits of all participating agents).

### III. RESULTS

In all results, the values are means (with error bars giving the 95% confidence interval) over 50 runs of the model from the same initial conditions, after  $10^9$  steps. The culture vector dimension is  $F = 5$ , the lattice linear dimension is  $L = 100$ , the von Neumann radius is 2 (hence the maximum number of participants in a public goods game is 13), and the marginal per capita return is fixed at  $\text{MPCR} = 0.6$ . Results for some

different values of  $L$  and  $\text{MPCR}$  are qualitatively similar [96]. The uncertainty constant in the Fermi update function is fixed at  $K = 0.1$  (results for some different values are shown in [96]), while the noise rate  $r = r'$ , the selection error and perturbation probability for culture vectors, is varied.

Figure 1 shows the fraction of conditional cooperators remaining after  $10^9$  steps as a function of  $q$  for different values of the noise rate ( $r = r'$ ). Cooperation only survives for zero noise, in which case the fraction of cooperators remaining increases with  $q$ , but not in a linear fashion. Even for noise rates as low as  $r = 10^{-6}$  cooperation dies out (although of course it is possible that for some noise rates  $0 < r < 10^{-6}$ , or for  $q > 100$ , cooperation may still survive).

Figure 2 shows the average number of participants in the public goods games after  $10^9$  steps. For low (or zero) noise rates, higher values of  $q$  result in fewer participants on average, and increasing noise results in nondecreasing average number of participants. When the noise rate is sufficiently large, however, the value of  $q$  becomes less important, until at noise rate  $10^{-4}$  or higher a similar average number of participants results for a given noise rate for all values of  $q$  greater than 30, and now decreases as the noise rate increases.

In summary, when the noise rate is zero, higher values of  $q$  result both in more conditional cooperators surviving and a smaller number of participants in the public goods games. However, when the noise rate is nonzero, cooperation dies out for any value of  $q$ , and if it is high enough the value of  $q$ , above a certain point, also becomes almost irrelevant to the number of participants.

Figure 3 shows the number of cultural regions after  $10^9$  steps as a function of  $q$ , for different noise rates. This quantity (the number of cultural regions) was used to measure diversity by Axelrod [1], and also in Flache and Macy [38]. An alternative measure is the largest region

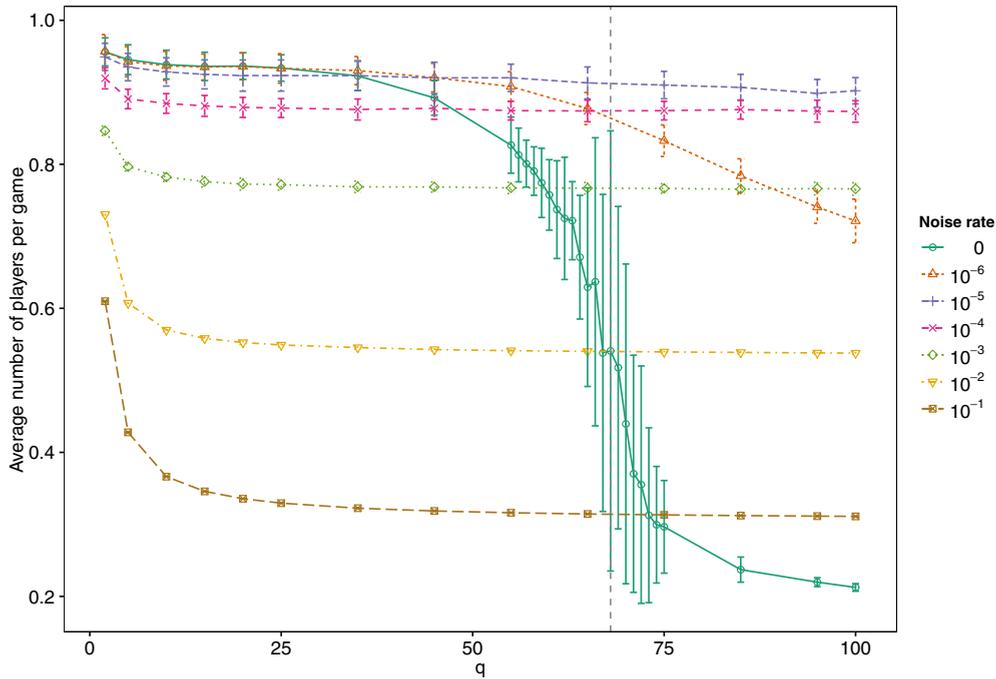


FIG. 2. Average number of players per public goods game after  $10^9$  steps as a function of  $q$  for different values of the noise rate. Lines are shown purely as an aid to the eye as  $q$  is an integer. The average number of players per game is normalized by division by the maximum possible number of players, which is the number of agents in the von Neumann neighborhood of the focal agent. The dashed vertical line shows the critical value of  $q$  (for zero noise).

size [32,38,91], which is an order parameter of the Axelrod model, separating the ordered (monocultural) phase from the disordered (multicultural) phase [34,91,97]. For zero noise rate, the phase transition of the Axelrod model where the regime switches from monocultural to multicultural [97] is

visible at the critical value of  $q = 68$  where the variance is at a maximum.

For low values of  $q$  and low enough noise rates, a monocultural (or nearly monocultural) regime results. At intermediate noise rates, a multicultural regime results for all

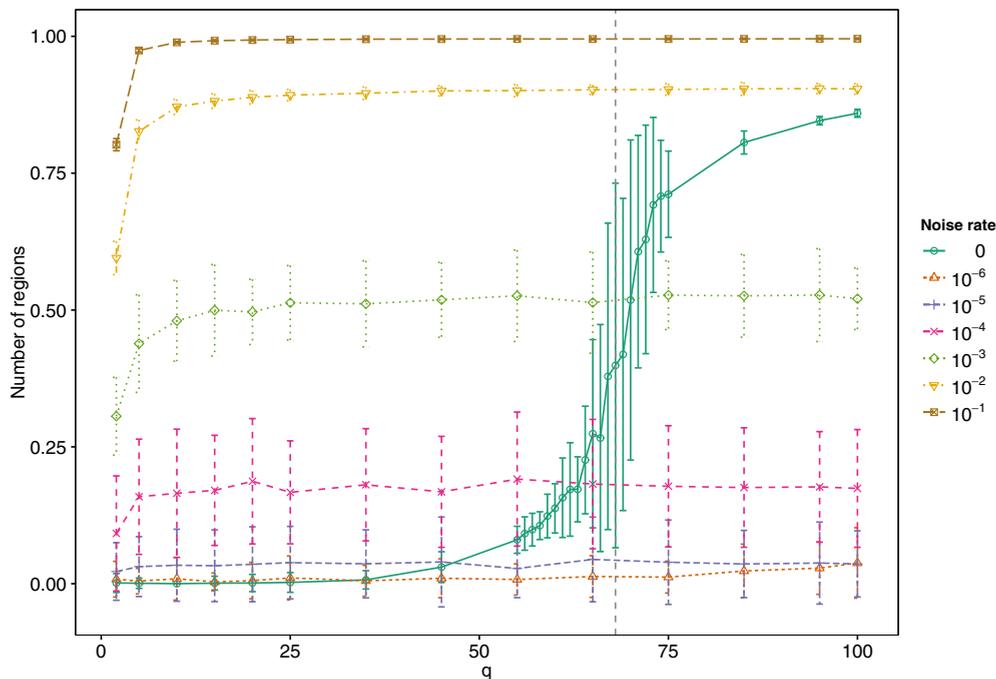


FIG. 3. Number of cultural regions (normalized by division by number of lattice sites) after  $10^9$  steps as a function of  $q$ , for different values of the noise rate. Lines are shown purely as an aid to the eye as  $q$  is an integer. The dashed vertical line shows the critical value of  $q$  (for zero noise).

values of  $q$  shown. As described by Flache and Macy [38], multilateral influence allows a stable multicultural regime to exist in the Axelrod model over a range of noise values, and this is also the case in our model (albeit for a much smaller range of noise values). For high enough values of  $q$ , stable monocultural regimes do not eventuate, as is well known for the Axelrod model [1,97, e.g.]. For very high noise rates, however, no matter what the value of  $q$ , a state of “anomie” prevails, in which stable cultural regions fail to form, as noted previously by Centola *et al.* [32], Flache and Macy [38], Mäs *et al.* [98].

The introduction of noise leads to qualitatively different results, a phenomenon first described in the Axelrod model by Klemm *et al.* [30]. Without noise, a phase transition at the critical value of  $q$  is apparent, at which the monocultural phase where cooperation dies out completely switches to a multicultural phase in which cooperation survives. However, when noise is introduced, cooperation never survives, and no phase transition occurs, with an almost constant number of cultural regions for each noise level, for any value of  $q > 15$ .

The conditions that lead to monoculture (or near monoculture), which are small values of  $q$  and low or zero noise rates, are conditions which lead to the extinction of cooperation. It would seem that, perhaps surprisingly, although conditional cooperators cooperate with probability proportional to their cultural similarity with others in the public goods game, a monocultural end point is actually inconsistent with the survival of cooperation in the model. In fact cooperation only survives for large values of  $q$  and zero (or possibly  $r < 10^{-6}$ ) noise rates: precisely conditions under which a multicultural regime forms. If the noise rate is too high, cooperation dies

out and cultural anomie results. If  $q$  is too small, cooperation again dies out, and a monocultural regime prevails.

Hence, cooperation and competition evolve together and affect each other, but not necessarily in a straightforward way. It is not simply a case of the level of cooperation being increased by cultural similarity acting as a tag, in which case we would expect that evolution towards a monocultural state would be accompanied by an increased level of cooperation. As previously discussed, Valori *et al.* [12] use two separate models for cooperation and cultural evolution, based on the explicit assumption that cultural evolution operates over a longer time scale. In our model, cooperation and cultural diffusion coevolve in the same model, and the different time scales emerge naturally. To see this, consider the time evolution of cooperation and cultural diffusion in the model. Figure 4 shows the time series of the average number of players per game, the fraction of agents which are cooperators, and the number of cultural regions, for three different values of  $q$ , for both zero noise and an intermediate noise rate. In most cases, the fraction of (conditional) cooperators reaches a stable value long before the cultural regions stabilize. This is particularly true for small values of  $q$ : for example, when  $q = 15$  (the top row of graphs in Fig. 4), the number of cooperators falls to zero rapidly, while the cultural regions continue to evolve for a much longer time. An exception is when the noise rate is zero and  $q$  is large, a condition in which the level of cooperation does not fall very far from its initial value (the bottom left graph of Fig. 4), in which both cooperation and cultural regions reach a frozen state very quickly. However, generally the level of cooperation stabilizes on an order of magnitude shorter time scale than the cultural regions. The middle row of graphs in

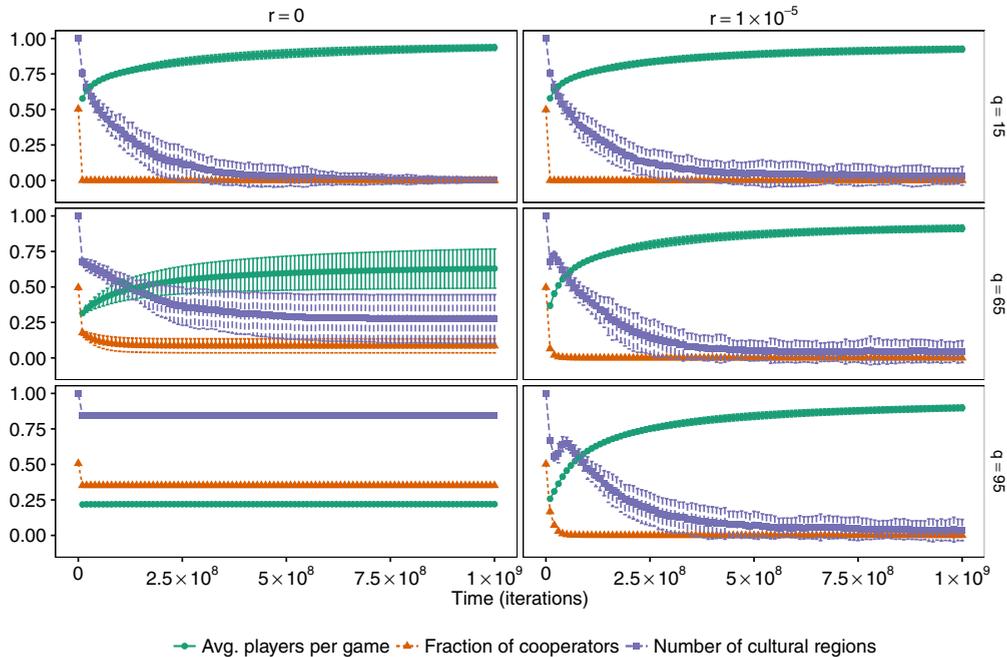


FIG. 4. Average number of players per game, fraction of agents which are (conditional) cooperators, and number of cultural regions, as a function of time, for three values of  $q$  and two noise rates (zero in the left column and intermediate in the right column). The average number of players per game is normalized by division by the maximum possible number of players, which is the number of agents in the von Neumann neighborhood of the focal agent.

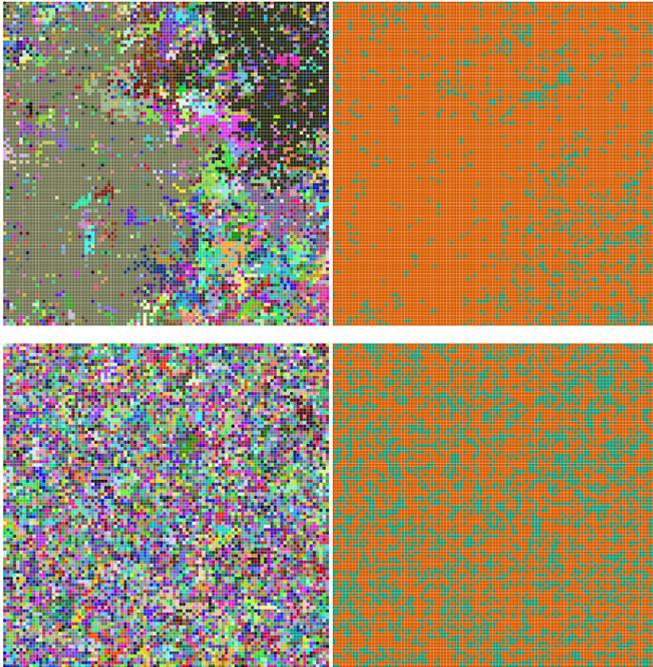


FIG. 5. Snapshot of the cultures (left) and cooperators (right) on the lattice after  $10^9$  iterations for  $r = 0$  and  $q = 65$ , top (corresponding to the end of the time series in the graph on the left of the central row in Fig. 4), and  $q = 95$ , bottom (corresponding to the end of the time series in the graph on the bottom left in Fig. 4). Each distinct culture vector is shown in a different color in the figures on the left, and in the figures on the right (conditional) cooperators are shown in green and defectors are shown as orange. The fraction of (conditional) cooperators for  $q = 65$  is approximately 11% and for  $q = 95$  is approximately 35%.

Fig. 4 is for a value of  $q$  near the critical value: it can be seen that when the noise rate is zero the variance of the number of regions and average number of players remains very high (less so for the fraction of cooperators, although still higher than for other values of  $q$ ).

Figure 5 shows the pattern of cultural regions and (conditional) cooperators on the lattice after  $10^9$  iterations for a single run with zero noise rate and two different values of  $q$ : near the critical value on the top and much higher than the critical value on the bottom. The top row of figures ( $q = 65$ ) shows a multicultural state, with one culture occupying a large area of the lattice, and many remaining smaller cultural regions. The bottom row of figures ( $q = 95$ ) shows another multicultural state, but this one where the very high level of initial diversity has resulted in an absorbing state being reached very quickly. In the top row of figures, it is apparent that the lattice positions where cooperators remain correspond to those cultures that are isolated, or on border regions, such that they can no longer interact. In the bottom row of figures, sizable cultural regions have not been able to form, and cooperators are still scattered over a large proportion of the lattice. For a small value of  $q$ , such as  $q = 15$  shown in the top left graph of Fig. 4, where a monocultural absorbing state is reached, cooperation dies out while interactions continue to occur.

Hence it is cultural dissimilarity that actually prevents agents from participating in public goods games with many

players and risking adopting the (dominant) strategy of defection. This explains the clearly observable inverse relationship between the average number of players (Fig. 2) and the number of cultural regions (Fig. 3): when the number of regions is small, there are few isolated cultures, and many agents in a given neighborhood can participate in public goods games based on cultural similarity. However, when the number of regions is larger, there are more isolated cultures or borders between incompatible cultures, and hence a larger number of agents who cannot participate in games where participation is conditional on cultural similarity.

#### IV. CONCLUSION

The results from this model, combining the Axelrod model of cultural dissemination with a spatial public goods game, and including both noise and multilateral social influence, suggest that cultural diversity can be beneficial for the evolution of cooperation. By making the probability of cooperation in a public goods game proportional to cultural similarity, we have found that, perhaps counterintuitively (or perhaps not, given the research on diversity and the promotion of cooperation discussed earlier), cultural evolution to a monocultural state is not associated with increased cooperation. In fact, quite the opposite: in no case does cooperation survive when a monocultural state is reached, but, rather, cooperation survives only when multicultural states are reached. Diversity, however, is not a sufficient condition for cooperation to survive: in many situations a multicultural state evolves after cooperation has died out completely. Furthermore, these results suggest that cultural information needs to be transmitted accurately in order to maintain both coherent cultural groups and cooperation.

Although it would appear that a greater scope of cultural possibilities in a model incorporating cultural homophily and social influence in a public goods game can lead to more conditional cooperators surviving, the conditions under which cooperation can survive in the long term are still very limited, and in no cases does the fraction of conditional cooperators increase from its initial level. “Strong ties” in a model with dynamic social networks increase cooperation in an iterated prisoner’s dilemma game [99], as does payoff-driven migration [21], and the combination of both migration and successful strategy imitation [8]. By incorporating coevolution of social networks and culture with migration [35], we would expect to see cooperation further promoted. Furthermore, although our model incorporates social influence with respect to culture, the updating rule is still pairwise, via the standard mechanism of the Fermi update rule. Incorporating “local influence,” in which the strategy is updated considering the average performance of each strategy in the entire neighborhood of the focal agent, into the update rule in the prisoner’s dilemma game gives qualitatively different results [100], which may also be the case if incorporated into our model.

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